A Low Complexity Wavelet OFDM Based on FPGA for Optical Communication Systems

Dang Le Khoa¹, Nguyen Thi Hong Thu¹, Nguyen Thanh Tu¹, Nguyen Huu Phuong¹, Hiroshi Ochi²

¹Faculty of Electronics and Telecommunications, HCM City University of Science, HCM City, Vietnam

²Department of Computer Science and Engineering, Kyushu Institute of Technology, Iizuka City, Japan

Abstract— This paper presents an implementation of a low complexity Wavelet OFDM system based on Haar function. The idea of low complexity is the arrangement and the multistage calculations of Wavelet transform. It was shown that the number of multiplier and adder was reduced. The system consists of Matlab simulation and FPGA-based implementation. Simulink and hardware models presented are scalable to higher speed allowing possible implementation in electronic processors for advanced optical communications.

Keywords - Low Complexity; Wavelet OFDM; FPGA; Haar function; Complex Wavelet.

I. INTRODUCTION

Optical communications have currently been advanced to deliver the highest bit rates ever imagined, the several hundred Gbits/s per optical wavelength channel [1][2]. This is possible due to the significant progresses in the use of coherent detection, orthogonal frequency division multiplexing (OFDM) technique, multiplexing of polarization modes of guided optical waves in single mode optical fibers, and the employment of ultra-high speed processing in the electronic domain. The advantages of OFDM have been well known and exploited to combat the intersymbol interference (ISI) in communication systems. The principal mechanism of OFDM is to generate parallel orthogonal channels in the frequency domain so that each subcarrier carries lower symbol rate. The orthogonal subcarriers allow an efficient use of the spectrum. However, this technique exists some drawbacks such as high PAPR and the use of Cyclic Prefix (CP). By using Wavelet Transform instead of the traditional FFT for OFDM system, these disadvantages are under control [3]. Haar wavelet transformation has been already proposed to improve the performance of communication systems at low signal- to-noise ratio (SNR)[4][5].

The discrete wavelet transform (DWT) and inverse discrete wavelet transform (IDWT) require many addition and multiplication operations. We need structures that can offer efficient generation and detection for hardware implementation of OFDM signals such as the architecture using lifting [6] and the derivation of the 9/7 wavelet filters [7]. FPGA offers the possibility of parallel structures and flexibility in developing prototypes [4][9][10].

In this paper, we propose a low complexity Wavelet OFDM based on FPGA for communication systems. We show the low complexity architecture for Haar DWT/IDWT and implement the OFDM transmitter and receiver based on the Stratix Development kit and associate software package DSP Builder of Altera.

This paper is organized as follows. Section 2 gives the essential features of Wavelet OFDM techniques. Section 3 briefly outlines the optical guided transmission media for the OFDM system. Simulink and hardware implementation of low complexity Wavelet OFDM based on FPGA is described in section 4. Section 5 presents the results obtained by simulink and FPGA-based hardware implementation. Finally, section 6 gives conclusions and future research.

II. WAVELET OFDM SYSTEMS

A. Haar wavelet transformation

Haar wavelet consists of a group of square waves with magnitude of ±1 in the interval [0,1) and the Haar scaling function is defined on the interval [0,1] as [11]

\[ \varphi(t) = \begin{cases} 
1, & \text{for } 0 \leq t < 1 \\
0, & \text{otherwise} 
\end{cases} \]  

(1)

The matrix of Haar wavelet transformation can be expressed as

\[ H_1 = 1 \]  

(2)

\[ H_{2N} = \frac{1}{\sqrt{2}} \left[ H_N \otimes [1,1] \right] \otimes [1,1] \]  

(3)

where \( I_N \) is the identity matrix. The \( \otimes \) is Kronecker product. A \( \otimes \) B is the matrix:

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\[ A \otimes B = \begin{bmatrix} a_{11}B & \cdots & a_{1n}B \\ \vdots & \ddots & \vdots \\ a_{m1}B & \cdots & a_{mn}B \end{bmatrix} \quad (4) \]

For example, \( H_S \) can be expressed as equation (5):

\[
H_S = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & -1 & -1 & -1 & -1 & -1 & -1 \\
1 & 1 & -1 & -1 & -1 & -1 & -1 & -1 \\
1/2 & 1/2 & 1/2 & 1/2 & 1/2 & 1/2 & 1/2 & 1/2 \\
0 & 0 & 0 & 0 & 1/2 & 1/2 & -1 & -1 \\
0 & 0 & 1/2 & 1/2 & 1/2 & -1 & 0 & \sqrt{2} \\
0 & 0 & 0 & 0 & 1/2 & 1/2 & -1 & \sqrt{2} \\
0 & 0 & 0 & 0 & 0 & \sqrt{2} & -1 & \sqrt{2} \\
\end{bmatrix} \quad (5) \]

B. Haar Wavelet OFDM

The principle for the Wavelet OFDM system is the combination of signal compositions to generate orthogonal subcarriers at transmitter. The composition uses IDWT. Likewise, the demultiplexing of the subcarriers at the receiver can be performed using DWT. The data at the transmitter \( X = [X_0, X_1, X_2, \ldots + X_{N-1}] \) is multiplied with a constant matrix \( H \).

In the case of the 8-subcarrier OFDM system, transmitter signal can be expressed as equation (6):

\[
\begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1/2 & 1/2 & 1/2 & 1/2 & 1/2 & 1/2 & 1/2 & 1/2 \\
1/2 & 1/2 & 1/2 & 1/2 & 1/2 & 1/2 & 1/2 & 1/2 \\
1/2 & 1/2 & -1 & -1 & -1 & -1 & -1 & -1 \\
1/2 & 1/2 & -1 & -1 & -1 & -1 & -1 & -1 \\
1/2 & 1/2 & 1/2 & 1/2 & 1/2 & 1/2 & 1/2 & 1/2 \\
0 & 0 & 1/2 & 1/2 & 1/2 & -1 & 0 & \sqrt{2} \\
0 & 0 & 0 & 0 & 1/2 & 1/2 & -1 & \sqrt{2} \\
0 & 0 & 0 & 0 & 0 & \sqrt{2} & -1 & \sqrt{2} \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix} \quad (6) \]

At the receiver, the signal is follow:

\[
\begin{bmatrix} (x_0 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7) \\ (x_0 + x_1 + x_2 + x_3 - x_4 - x_5 - x_6 - x_7) \\ 1/2 (x_0 + x_2 - x_3) \\ 1/2 (x_4 + x_5 - x_6 - x_7) \\ 1/2 (x_0 - x_1) \\ 1/2 (x_2 - x_3) \\ 1/2 (x_4 - x_5) \\ 1/2 (x_6 - x_7) \end{bmatrix} = \frac{1}{\sqrt{8}} \begin{bmatrix} 1 \/ \sqrt{8} \\
(x_0 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7) \\
(x_0 + x_1 + x_2 + x_3 - x_4 - x_5 - x_6 - x_7) \\ 1/2 (x_0 + x_2 - x_3) \\ 1/2 (x_4 + x_5 - x_6 - x_7) \\ 1/2 (x_0 - x_1) \\ 1/2 (x_2 - x_3) \\ 1/2 (x_4 - x_5) \end{bmatrix} \quad (7) \]

III. OPTICAL TRANSMISSION MEDIA

The fundamental impairments of optical fiber are considered such as nonlinear effects, attenuation and distortion. The propagation of an optical carrier-modulated signal can be represented by the non-linear Schrodinger wave equation [12]:

\[ \frac{\partial A}{\partial z} = -\frac{\alpha}{2} A - \beta_1 \frac{\partial A}{\partial t} - j \beta_2 \frac{\partial^2 A}{\partial t^2} + \frac{1}{6} \beta_3 \frac{\partial^4 A}{\partial t^4} - j \gamma |A|^2 A \quad (8) \]

where the amplitude \( A = A(z,t) \) is the complex envelope carried by the lightwaves, along the propagation \( z \) axis; \( \alpha \) accounts for attenuation; \( \beta_1 \) indicates differential group delay (DGD); \( \beta_2 \) and \( \beta_3 \) represent second- and third-order dispersion factor of fiber CD; \( \gamma \) is the nonlinear coefficient[9].

A single fiber transmission span consists of a Single Mode Fiber (SMF), an optical amplifier EDFA (Figure 1.).

![Figure 1](image-url)  
Figure 1. Single fiber transmission span

We simulate an optical communications link over several hundred kilometers by cascading these spans from one end of the transmission link to another. The loss of each span is compensated by an EDFA.

The NES is regarded as the propagation equation of an optical pulse in single mode fiber. The numerical approach which is used to figure out the nonlinear Schrodinger equation is known as the Split-Step Fourier Method (SSFM). We use the symmetric SSFM to solve equation 8 approximately as follows[13]:

\[ A(z+h,t) \approx \exp \left( \frac{h}{2} \hat{D} \right) \exp \left\{ j N[A] + \frac{h}{2} \hat{B} \right\} \quad (9) \]

Where \( \hat{D} = -j(\beta''/2) \frac{\partial^2}{\partial t^2} \) is the dispersion operator and \( \hat{N}[A] = j \gamma |A|^2 \) is nonlinear operator.
The accuracy and efficiency of this method depend on the distribution of step sizes along fiber and on both time and frequency domain resolutions. Finding an optimal step is not easy and depends on particular optical system. It is beyond our study. The accuracy could be improved among total number of steps. To be practical, the step size we choose in the simulation is 100 meters in each span which is 80 km long. The long haul fiber communication link in this simulation is simulated by cascading many single spans. The Figure 2. is Simulink model of a 800 km fiber long which is formed from 10 single spans.

EDFAs will overcome the attenuation on fiber optic. At the receiver the dispersion of the transmission medium on all sub-carrier channels of the OFDM symbol is eliminated using the following equation [14]:

\[
\phi = \frac{1}{2} \beta_2 \omega^2 L
\] (10)

\[
\beta_2 = -\frac{\lambda^2}{2\pi c} D
\] (11)

where \( \beta_2 \) is group velocity dispersion, \( D \) is fiber dispersion, \( L \) is the fiber length, and \( \omega \) is the optical frequency.

IV. LOW COMPLEXITY HAAR WAVELET OFDM DESIGN

A. Low Complexity Haar Wavelet

For the case of an 8-subcarrier OFDM, equation (6) shows that we need 4 multiplications and 3 additions for each sample of \( x(k) \). Hence to obtain a complete set of IDWT coefficients, we need 32 multiplications and 24 additions. This paper proposes a low complexity algorithm. The inverse discrete wavelet transformation is implemented by 4 steps.

Step 1: multiply each \( X(k) \) with a constant

\[
a_0 = \frac{1}{\sqrt{8}} X_0, \quad a_1 = \frac{1}{\sqrt{8}} X_1, \quad a_2 = \frac{1}{2} X_2, \quad a_3 = \frac{1}{2} X_3
\]

\[
a_4 = \frac{1}{\sqrt{2}} X_4, \quad a_5 = \frac{1}{\sqrt{2}} X_5, \quad a_6 = \frac{1}{\sqrt{2}} X_6, \quad a_7 = \frac{1}{\sqrt{2}} X_7
\] (12)

Step 2: calculate \( g_0^1 \) and \( h_0^1 \)

\[
g_0^1 = a_0 + a_1, \quad h_0^1 = a_0 - a_1
\] (13)

Step 3: calculate \( g_0^2, h_0^2, g_1^2, \) and \( h_1^2 \)

\[
g_0^2 = g_0^1 + a_2, \quad h_0^2 = h_0^1 - a_2
\]

\[
g_1^2 = g_1^1 + a_3, \quad h_1^2 = h_1^1 - a_3
\] (14)

Step 4: calculate the transmitter signal

\[
x_0 = g_0^2 + a_4, \quad x_1 = g_0^2 - a_4
\]

\[
x_2 = h_0^2 + a_5, \quad x_3 = h_0^2 - a_5
\]

\[
x_4 = g_1^2 + a_6, \quad x_5 = g_1^2 - a_6
\]

\[
x_6 = h_1^2 + a_7, \quad x_7 = h_1^2 - a_7
\] (15)

We need 8 multiplications, 7 additions and 7 subtractions for a complete set of IDWT coefficients as the Figure 3.
Figure 3. Low complexity Haar IDWT

The data at the receiver is \( x = [x_0, x_1, \ldots, x_7] \).
Likewise, the discrete Haar wavelet transformation is implemented by 4 steps. The architecture of low complexity Haar DWT is shown in Figure 4.

Step 1: calculate \( G_0^1, G_1^1, G_2^1, G_3^1, H_0^1, H_1^1, H_2^1 \), and \( H_3^1 \)

\[
\begin{align*}
G_0^1 &= x_0 + x_1, H_0^1 = x_0 - x_1 \\
G_1^1 &= x_2 + x_3, H_1^1 = x_2 - x_3 \\
G_2^1 &= x_4 + x_5, H_2^1 = x_4 - x_5 \\
G_3^1 &= x_6 + x_7, H_3^1 = x_6 - x_7
\end{align*}
\]  

Step 2: calculate \( G_0^2, G_1^2, H_0^2 \) and \( H_2^2 \)

\[
\begin{align*}
G_0^2 &= G_0^1 + G_1^1, H_0^2 = G_0^1 - G_1^1 \\
G_1^2 &= G_2^1 + G_3^1, H_1^2 = G_2^1 - G_3^1
\end{align*}
\]  

Step 3: calculate the data

\[
\begin{align*}
a_0 &= G_0^2 + G_1^2, a_1 = G_0^2 - G_1^2 \\
a_2 &= H_0^2, a_3 = H_1^2 \\
a_4 &= H_1^2, a_5 = H_1^1 \\
a_6 &= H_1^1, a_7 = H_1^1
\end{align*}
\]  

Step 4: multiply each \( X(k) \) with a constant

\[
\begin{align*}
a_0 &= \frac{1}{\sqrt{8}} a_0, X_1 = \frac{1}{\sqrt{8}} a_1 \\
a_2 &= \frac{1}{2} a_2, X_3 = \frac{1}{2} a_3 \\
a_4 &= \frac{1}{\sqrt{2}} a_4, X_5 = \frac{1}{\sqrt{2}} a_5 \\
a_6 &= \frac{1}{\sqrt{2}} a_6, X_7 = \frac{1}{\sqrt{2}} a_7
\end{align*}
\]
B. OFDM Systems Design

The OFDM transmission system FPGA-based platform is shown in Figure 5, which consists of IQ mapper/IQ demapper, serial to parallel converter (S/P), IDWT/DWT, parallel to serial parallel converter (S/P).

Data used for inspection of the system generating the randomizer is stored in the RAM memory. SignalTap of Altera FPGA interfaced via the Standard Joint Test Action group (JTAG) is used for data transmission of the system to computer. Digital signals are converted to analog form via the DAC. The system process signals at the baseband signal, thus the signal spectrum is evaluated on the I- and Q-components.

The software platform used in this work is the DSP Builder of Altera operating on MATLAB Simulink environment.

IQ mapper is the modulation technique to transform the sequence of m bits into a constellation. The number of bits m dictates the number of states of the constellation. For example, a BPSK with 1 bit per symbol has 2 points on the constellation. The mapper uses look-up tables for I- and Q-components (Figure 6).

Figure 4. Low complexity Haar DWT

Figure 5. Hardware experimental of the OFDM systems.
S/P block converts the serial bit sequence to parallel to assembly the OFDM symbol in frequency domain. Each symbol represents a frequency spectrum to superimpose on the subcarriers. In this design, OFDM symbol is defined as a set of 8 subcarrier channels whose number determines the number of input of the IDWT and DWT. These signals are converted to analog via DACs and then launched into the optical domain. We assume that the channel is Additive white Gaussian noise.

At the receiving end, ADCs convert analog signals into digital signals before taking the Wavelet transform. The constellation demodulator must set the decision levels so as to determine the constellation points of the receiver. The decision point is based on the shortest Euclidean distance to the received signals. When BPSK modulation is used, the demapper can simply be determined by evaluating the most significant bit of the received bit sequence which indicates the sign bit.

C. Simulation and Hardware Integrated Platform of Optical WOFDM System

The overall system is shown in Figure 7. Transmitter has two main functions. The first block is a block ODFM, which is supposed to create the OFDM signal in electrical domain. The second block is the Mach-Zehnder external modulation of electrical signals into optical signals corresponding to the two components I - Q. The Q is phase shifted 90 degrees. The signals are combined and launched into the fiber.

Output waveform when the signal is transmitted over optical fibers is obtained by solving equations (8). There are many methods to solve this equation, the common approach is to use a split-step Fourier method. The idea of this method is to divide the fiber into smaller sections with a length of about 200m to 500m. On the small stage, assuming that the effect of linear and nonlinear effects are independent of each other.

The receiver is responsible for converting signals from optical to electrical. In particular, local oscillator frequency LO created equal frequency of the transmitter laser. Optical signal to the receiver is separated into two components I, Q. Which go to the balanced receiver. The structure of the balanced receiver includes two photodetectors. The two photodetectors will increase 3 dB gain compared to the detector only a photodetector. Electrical signal will be put in OFDM receiver. This block has the function to do the opposite steps at the transmitter to receive transmitted bit sequence.
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V. SIMULATION AND EXPERIMENTAL PLATFORM RESULTS

A. Complexity of IFFT/FFT and IDWT/DWT

The Complexity of IFFT/FFT and IDWT/DWT for the 8-subcarrier OFDM system employing are shown in TABLE I.

<table>
<thead>
<tr>
<th>TABLE I. THE COMPLEXITY OF IFFT/FFT AND IDWT/DWT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radix 2 FFT/IFFT</td>
</tr>
<tr>
<td>--------------------</td>
</tr>
<tr>
<td>Multiplications</td>
</tr>
<tr>
<td>Additions</td>
</tr>
<tr>
<td>Subtractions</td>
</tr>
</tbody>
</table>

B. Hardware Integrated Platform

The OFDM signals were monitored by the software platform SignalTap integrated in the hardware system. The results were displayed on a computer.

In order to study the functions and performances of each block of the system, we monitored and accumulated data at the input and output of each block. For example, Figure 8 shows the waveform after Mapper and Figure 9 shows the waveform after P/S respectively.

The speed of the operating system was set at 80 MHz for the complete OFDM system. The number of DWT points was 8. The system employed BPSK modulation scheme with 1 bits/symbol. Thus the useful speed of the system was 80Mb/s.

FPGA was used for the design of Wavelet OFDM system. The details of the resources are listed in TABLE II.

TABLE II. RESOURCES OF THE WAVELET OFDM SYSTEM

<table>
<thead>
<tr>
<th>Device</th>
<th>EP1S2SF780C5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total logic elements</td>
<td>2,539 / 25,660 (10%)</td>
</tr>
<tr>
<td>Total memory bits</td>
<td>487,823 / 1,944,576 (25%)</td>
</tr>
<tr>
<td>DSP block 9-bit elements</td>
<td>4 / 80 (5%)</td>
</tr>
<tr>
<td>Total PLLs</td>
<td>1 / 6 (17%)</td>
</tr>
<tr>
<td>Total DLLs</td>
<td>0 / 2 (0%)</td>
</tr>
</tbody>
</table>

The BER performance of DWT OFDM system using BPSK modulation format is shown in Figure 10. The figure shows that the performance of Low Complexity IDWT/DWT matched with the performance of direct IDWT/DWT and improved by 1.25dB as compared with IFFT/FFT.

![Figure 10. BER of DWT OFDM system versus SNR](image)

C. Simulink of Coherent Optical WOFDM System

OFDM signal is modulated by the Mahnch-Zender (MZ) modulators for transmission on optical fiber. For optical guided wave channel the distortion is mainly due to chromatic and polarization dispersion effects and nonlinear self phase modulation effects [2]. The receiver uses two optical coherent detectors which serve as an optical-to-electrical OFDM I/Q converter before being sampled by the ADC.

The system is demonstrated for a transmission with dispersion compensation at 10Gb/s. We apply commonly used system parameters for our simulation in TABLE III.

<table>
<thead>
<tr>
<th>TABLE III. FIBER AND EDFA PARAMETERS FOR SINGLE SPAN</th>
</tr>
</thead>
<tbody>
<tr>
<td>SMF</td>
</tr>
<tr>
<td>Loss factor dB/km</td>
</tr>
<tr>
<td>Dispersion coeff. D = 17 (ps/nm.km)</td>
</tr>
<tr>
<td>Nonlinear coeff. L = 80 km</td>
</tr>
<tr>
<td>NF = 5</td>
</tr>
</tbody>
</table>

![Figure 11. BER versus OSNR](image)
Figure 11. shows that system BER versus OSNR of SSMF with dispersion compensation at the optimal optical launch power. The optimal optical launch power is about 10dBm.

VI. CONCLUDING REMARKS

In this paper, we proposed a low complexity Wavelet OFDM based on FPGA for optical communication systems. The principle blocks of system consisted of Wavelet OFDM symbols, mapping to BPSK symbols, models of transmission medium. The proposed model needs 16 multiplications, 14 additions, and 14 reductions for 8 IDWT/DWT pair transformation. The models presented in this paper are currently modified to combat peak to average power ratio (PAPR). These works will be reported in the future.

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REFERENCES


