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Bilateral Decision Making Under Asynchronous Time Constraint

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***Abstract*: Decision making is indispensable to any optimization in intelligent computing. In this study, we discuss a mathematical foundation for bilateral decision making under synchronous and asynchronous time constraint. The problem on time constraint is to evaluate the cost of time or the value of the entire duration of certain decision process. We propose a formula to compute the cost of time by introduction of opportunity cost to its evaluation. We also propose two formulas on the strategic points to minimize the cost of time for decision process. under synchronous or common time constraint and asynchronous or uncommon time constraints. The proposed formulas contribute to accelerating time-sensitive decision making, instead of the heuristic point of the half time.**

***Keywords*:** optimization, intelligent computing, decision making, cost of time, time constraint, asynchronous.

I. Introduction

In this study, we discuss a mathematical foundation for bilateral decision making under synchronous and asynchronous time constraint. Decision making is indispensable to any optimization in intelligent computing. A generally accepted point for decision making is *half time* or the half entire duration of decision process as a heuristic point for decision making. The concept of *cost of time* or value of the entire duration of certain decision process allows us to locate a *strategic point* or a point or timing for decision making to minimize the cost of time. Strategic points accelerate time-sensitive decision making, instead of the heuristic point of half time.

The community of optimization decision making in a variety of implementations, *e.g.*, human-machine intelligent systems for decision making [18, 29, 2, 5, 11]. Not a large number of researches have examined decision making under *time constraint* or time stress [16]. Time constraint is, however, discussed in information search strategies [30] and real-time decision making of multi-agents [15]. We have also discussed bilateral decision making under synchronous time constraint in our introductory studies [21, 22, 23, 24]. This study is motivated to give a general formula on bilateral decision making under not only synchronous but also asynchronous time constraint [25, 26]. The problem on time constraint is to evaluate the cost of time as the value of the entire duration of certain decision process [20]. Its well-known concept of computation is *opportunity cost* which is the value of the next-best alternative use of that time [19]. We propose a formula to compute the cost of time under both synchronous and asynchronous time constraint by introduction of opportunity cost to its evaluation.

Opportunity cost allows us to discuss strategic points under time constraint from the viewpoint of not heuristics but rationality. The subjects to time constraint rather than other factors often choose irrational strategies in their decision making [14]. A typical irrationality under time constraint is found in *a priori* acceptance of the heuristic point of half time, but not a strategic point for decision making to minimize cost of time. A number of solutions for optimization of decision making explicitly or implicitly include the heuristic point in their constraint management mechanisms [27].

Instead of the heuristic point, we propose two formulas on strategic points under synchronous and asynchronous time constraint. First, we propose a formula on strategic points under *synchronous* or common time constraint among decision makers. The strategic points under synchronous time constraint are always located at the one-third entire duration and the one-third remaining duration of decision process, instead of the heuristic point of half time. Second, we propose a formula on strategic points under *asynchronous* or uncommon time constraint among decision makers. Strategic points under asynchronous time constraint are located at the one-third shorter entire duration and/or at the one-third shorter remaining duration or at the one-third longer entire duration of decision process in certain specific cases, *i.e.*, specific ranges of the ratios of shorter duration to longer duration of decision process.

We give brief definitions to following concepts which are introduced in this study, as below: First, decision makers in *bilateral decision making* have only two options to leave from or to stay in decision process [6]; Second, bilateral decision making has two types of games, among which a *repeated game* consists of some number of repetitions of some base game but a *single stage game* is a non-repeated game [12]; Third, the value of *gain* in a game is subject to a certain hypothetical wage rate of possible works which are taken by decision makers instead of their current tasks [1]. The hypothetical wages are substitutable with prices of the individual items which are available as objects of transaction or negotiation during certain decision process [3]. The gain takes a form of linear curve on its function [13]; And, Fourth, *free*

access to information regarding any gain or the prices of individual items in decision process is indispensable to the computation of cost of time [3].
The cost of time in a repeated game is computed on the basis of opportunity cost, because a single stage game always takes unique values on prices of its individual items, however, a repeated game does not [19]. Figure 1 describes the basic idea on the computation of opportunity cost in a repeated game by using the prices of individual items in a single stage game: Suppose that Group A gained \$100 at 50 minutes during its entire 150 minutes in a repeated game; Group B gained \$200 at 100 minutes during its entire 200 minutes in a single stage game; Thus, Group A dismissed its opportunity of the larger gain, here \$200, so that its remaining time, here 100 minutes, was evaluated as equivalent to the same amount of that monetary value, \$200, as its opportunity cost. Thus, any op-

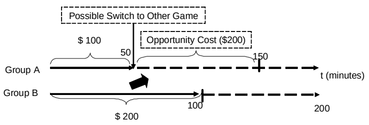

Figure. 1: The computation of opportunity cost

portunity cost of time for acquiring certain gain is equivalent to the additive of prices of individual items which are available during certain decision process.
We rely on following findings in behavioral sciences, especially, cognitive science, which are introduced as the limitations to this study, as below: First, we introduce the findings in *transitional games* or the games of the decision making under time constraint. Decision makers under time constraint do not always behave rationally in transitional games, but behave more like real people to sometimes select irrational strategies [4, 17, 7]; Second, decision makers under time stress are risk averse or conservative at lower risk levels [8]; And, Third, people under time stress face only two outcomes, *i.e.*, all or nothing, here, leave or stay, and are always to account for certain equilibrium across any stage of their games [9].
We also apply an assumption on *reinforcement learning* as a limitation to this study, as below: Decision makers are to be aware of the information regarding the works of their environments, such as the gain and the cost of time through their first decision making, however, they are to only carry it out to their succeeding or future decision process [28].
This study contributes to the computation of cost of time and the formulation of strategic points under both synchronous and asynchronous time constraint, instead of the heuristic point. Especially, the proposed formulas are to minimize cost of time and to accelerate time-sensitive decision making which is indispensable to any solutions for optimization in intelligent computing.
The remaining of this study is organized as follows. In Section II, we give a formula to evaluate cost of time, and deduct from it two formulas on strategic points under synchronous and asynchronous time constraint, respectively. In Section III, we conduct a feasibility check on those formulas in their applications to a case study. In Section IV, we give various analyses of the case study with the contributions and limitations of the proposed formulas. In Section V, we conclude this study with our future work.

II. Formula

In this section, we give a formula to evaluate cost of time, and then deduct from it two formulas on strategic points to minimize cost of time for decision process under synchronous and asynchronous time constraint, respectively.

A. Cost of Time

Any cost of time is equivalent to the additive of prices of individual items which are available during certain decision process. Here, we simply define the cost of time under synchronous time constraint, though its definition is applicable to the cost of time under asynchronous time constraint without any limitation. We introduce two assumptions, as below:

Assumption 1 *Any price of an individual item is always given as a certain static value in a single stage game;*

Assumption 2 *Any cost of time is in proportion to the ratio of elapsed time to the entire duration of certain decision process.*

Assumptions 1 and 2 assure that prices of individual items in single stage games and the entire duration of decision process are given as static values, respectively.
We give a formula to evaluate cost of time in single stage games as $C^s_{(t)}$, as below:

Definition 1

$$C^s_{(t)} \equiv \frac{t}{\tau} \cdot \sum_{k=1}^{n} p_k. \qquad (1)$$

s.t. t *represents elapsed time in certain decision process,* $t \in \mathcal{R}$*;* τ *is given as a certain static value to the entire duration of certain decision process,* $\tau \in \mathcal{R}$*;* p_k *represents a price of the* k*-th individual item from* 1 *to* n *in a single stage game,* $p \in \mathcal{R}$*,* k*,* $n \in \mathcal{N}$*.*

Definition 1 assures that any cost of time is equivalent to the gain or the additive of prices of individual items which are available in single stage games. The gain in a repeated game does not have its unique value in a variety of its next-best alternatives so that we use opportunity cost for its computation to identify the unique value.
We introduce two more assumptions, as below:

Assumption 3 *Any repeated game is to be expected to spend at least the same duration of its previous game in its next game;*

Assumption 4 *Any function on the cost of time always takes a certain equivalent value in its transition from a repeated game to a single stage game.*

Suppose that a repeated game transits into a single stage game at the half time or the half entire duration; The new single stage game takes that same half time again so that both the repeated game and the single stage game face a certain equivalent value at the half time, $t = \frac{\tau}{2}$. Assumptions 1 to 4 allow us to deduct a lemma on the gain in a repeated game which is to be a certain static value, P^r.

Lemma 1

$$P^r = \frac{1}{2}\sum_{k=1}^{n} p_k. \tag{2}$$

$$\frac{\frac{\tau}{2}}{\tau}\sum_{k=1}^{n} p_k = \lfloor \frac{\tau}{\frac{\tau}{2}} \rfloor \cdot \frac{\frac{\tau}{2}}{\tau} \cdot P^r. \quad \because C^s_{(\frac{\tau}{2})} = C^r_{(\frac{\tau}{2})}.$$

s.t. $\lfloor \cdot \rfloor$ is the floor function; $\lfloor \frac{\tau}{t} \rfloor$ represents the possible times of repetitions of some base game to acquire the next-best alternatives as opportunity cost.

We give a formula to evaluate cost of time in repeated games using the lemma 1 as $C^r_{(t)}$, as below:

Definition 2

$$C^r_{(t)} \equiv \lfloor \frac{\tau}{t} \rfloor \cdot \frac{t}{\tau} \cdot \frac{1}{2}\sum_{k=1}^{n} p_k. \tag{3}$$

B. Strategic Point Under Synchronous Time Constraint

Here, we give a formula on strategic point under synchronous time constraint. Suppose that a repeated game transits into a single stage game at the heuristic point of half time, discussed as above. Decision makers in this game *a priori* accept half time as their point for decision making. In this case, we give a function on the ratio of gain to cost of time, $\frac{P}{C_{(t)}}$, as below:

Definition 3

$$\frac{P}{C_{(t)}} = \begin{cases} \frac{P^r}{C^r_{(t)}} = \frac{2\tau}{\lfloor \frac{\tau}{t} \rfloor \cdot t} & \textit{if} \quad 0 \leq \frac{t}{\tau} \leq \frac{1}{2}, \\ \frac{P^s}{C^s_{(t)}} = \frac{\tau}{t} & \textit{else if} \quad \frac{1}{2} \leq \frac{t}{\tau} \leq 1. \end{cases} \tag{4}$$

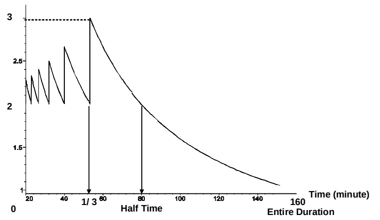

Figure. 2: The function on the ratio of gain to cost of time, $\frac{P}{C_{(t)}}$: a repeated game transits into a single stage game at half time

Figure 2 describes that the function always takes its largest value 3.00 at the one-third entire duration of decision process before the heuristic point, half time. Its peak is the strategic point to minimize cost of time. We prove this finding as a theorem, as below:

Theorem 1

$$\arg\max_{[0 \leq t \leq \frac{\tau}{2}]} \frac{P}{C_{(t)}} = \lim_{t \to \frac{\tau}{3}+0} \frac{P^r}{C^r_{(t)}} = 3.00. \tag{5}$$

$$\arg\max_{[0 \leq t \leq \frac{\tau}{2}]} \frac{P}{C_{(t)}} = \lim_{t \to \frac{\tau}{3}+0} \frac{P^r}{C^r_{(t)}} = 2 \cdot \frac{\tau}{\lfloor \frac{\tau}{(\frac{\tau}{3}+0)} \rfloor \cdot \frac{\tau}{3}} = 3.00$$

$$> \lim_{t \to \frac{\tau}{4}+0} \frac{P^r}{C^r_{(t)}} = 2.67 > \lim_{t \to \frac{\tau}{5}+0} \frac{P^r}{C^r_{(t)}} = 2.50 > \cdots.$$

$$\because \lim_{t \to \frac{\tau}{3}+0} \lfloor \frac{\tau}{t} \rfloor = 2; \lim_{t \to \frac{\tau}{4}+0} \lfloor \frac{\tau}{t} \rfloor = 3; \lim_{t \to \frac{\tau}{5}+0} \lfloor \frac{\tau}{t} \rfloor = 4; \cdots.$$

Therefore, any strategic point under synchronous time constraint is always located at the one-third entire duration of decision process before the heuristic point of half time. After half time, decision makers have two options: leave from or stay in the current decision process. The former option allows them in the new process to scale down by half on the duration of their decision process and to apply the function on the ratio of gain to cost of time as described in Equation (4). In this option, its function takes the following equation:

Definition 4

$$\frac{P}{C_{(t)}} = \begin{cases} \frac{P^r}{C^r_{(t)}} = \frac{\tau}{\lfloor \frac{\tau}{2t-\tau} \rfloor \cdot (t-\frac{\tau}{2})} & \textit{if} \quad \frac{1}{2} \leq \frac{t}{\tau} \leq \frac{3}{4}, \\ \frac{P^s}{C^s_{(t)}} = \frac{1}{\frac{2t}{\tau}-1} & \textit{else if} \ \frac{3}{4} \leq \frac{t}{\tau} \leq 1. \end{cases} \tag{6}$$

The upper of Figure 4 describes that the function in the second repeated game always takes its largest value 3.00 at the two-thirds entire duration or the one-third remaining duration of decision process. Its peak is a strategic point to minimize cost of time.

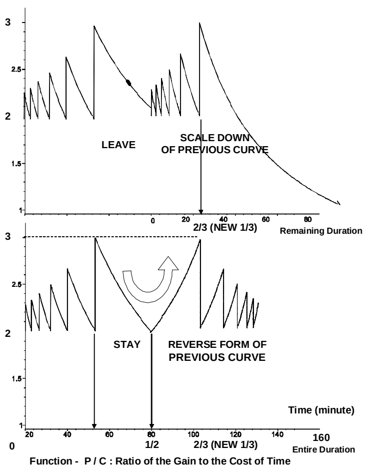

Figure. 3: Strategic points after leave and stay

On the contrary, decision makers select the strategy to stay in the current decision process and face two types of functions: One function is the same with the above equation (6); The other function takes reverse or backward move from half time to the point of the one-third entire duration of decision process. In this option, its function takes the following equation:

Definition 5

$$\frac{P^r}{C^r_{(t)}} = \frac{2\tau}{\lfloor \frac{\tau}{\tau-t} \rfloor \cdot (\tau - t)} \ (\frac{1}{2} \leq \frac{t}{\tau} \leq 1). \tag{7}$$

The lower of Figure 4 describes that the function in the second repeated game always takes the largest value 3.00 at the two-thirds entire duration or the one-third remaining duration of decision process. Its peak is a strategic point to minimize cost of time. We prove this finding as a theorem, as below:

Theorem 2

$$\arg\max_{[\frac{1}{2}\leq t\leq 1]} \frac{P}{C_(t)} = \lim_{t\to\frac{2\tau}{3}+0} \frac{P^r}{C^r_{(t)}} = \lim_{t\to\frac{2\tau}{3}-0} \frac{P^r}{C^r_{(t)}} = 3.00. \tag{8}$$

$$\arg\max_{[\frac{1}{2}\leq t\leq 1]} \frac{P}{C_(t)} = \lim_{t\to\frac{2\tau}{3}+0} \frac{P^r}{C^r_{(t)}}$$

$$= \frac{\tau}{\lfloor \frac{\tau}{2(\frac{2\tau}{3}+0)-\tau} \rfloor \cdot (\frac{2\tau}{3}-\frac{\tau}{2})} = 3.00$$

$$> \lim_{t\to\frac{5\tau}{8}+0} \frac{P^r}{C^r_{(t)}} = \frac{8}{3} = 2.67 > \cdots$$

$$> \lim_{t\to\frac{3\tau}{5}+0} \frac{P^r}{C^r_{(t)}} = \lim_{t\to\frac{3\tau}{5}+0} \frac{P^s}{C^s_{(t)}} = \frac{5}{2} = 2.50;$$

$$\arg\max_{[\frac{1}{2}\leq t\leq 1]} \frac{P}{C_(t)} = \lim_{t\to\frac{2\tau}{3}-0} \frac{P^r}{C^r_{(t)}}$$

$$= \frac{2\tau}{\lfloor \frac{\tau}{\tau-(\frac{2\tau}{3}-0)} \rfloor \cdot (\tau-\frac{2\tau}{3})} = 3.00$$

$$> \lim_{t\to\frac{5\tau}{8}-0} \frac{P^r}{C^r_{(t)}} = \frac{8}{3} = 2.67 > \cdots$$

$$> \lim_{t\to\frac{3\tau}{5}-0} \frac{P^r}{C^r_{(t)}} = \lim_{t\to\frac{3\tau}{5}-0} \frac{P^s}{C^s_{(t)}} = \frac{5}{2} = 2.50.$$

$$\because \lim_{t\to\frac{2\tau}{3}+0} \lfloor \frac{\tau}{2t-\tau} \rfloor = \lim_{t\to\frac{2\tau}{3}-0} \lfloor \frac{\tau}{\tau-t} \rfloor = 2.00.$$

Therefore, another strategic point under synchronous time constraint is always located at the one-third remaining duration of decision process after the first heuristic point of half time and before the other heuristic point of half time in both single stage games and repeated games.

C. Strategic Point Under Asynchronous Time Constraint

Here, we give a formula on strategic point under asynchronous time constraint. Suppose that each decision maker has his or her individual uncommon entire duration of decision process. We introduce an assumption for simplification of the computation on cost of time under asynchronous time constraint, as below:

Assumption 5 *Any synchronization of the respective duration of decision process is not to be expected or executed during any process of decision making.*

Assumption 5 assures that the function on the ratio of gain to cost of time under asynchronous time constraint takes the following equation on the basis of Equations (4), (6) and (7), $\frac{P^a}{C^a_{(t)}}$:

Definition 6

$$\frac{P^a}{C^a_{(t)}} = \frac{1}{2} \sum_{\tau\in\tau^1,\tau^2} \frac{P^a_\tau}{C^a_{\tau(t)}} = \begin{cases} \frac{2\tau}{\lfloor\frac{\tau}{t}\rfloor\cdot t} & (0\leq\frac{t}{\tau}\leq\frac{1}{2}), \\ \frac{\tau}{\lfloor\frac{\tau}{2t-\tau}\rfloor\cdot(t-\frac{\tau}{2})} & (\frac{1}{2}\leq\frac{t}{\tau}\leq\frac{3}{4}), \\ \frac{1}{\frac{2t}{\tau}-1} & (\frac{3}{4}\leq\frac{t}{\tau}\leq 1), \\ \frac{2\tau}{\lfloor\frac{\tau}{\tau-t}\rfloor\cdot(\tau-t)} & (\frac{1}{2}\leq\frac{t}{\tau}\leq 1). \end{cases} \tag{9}$$

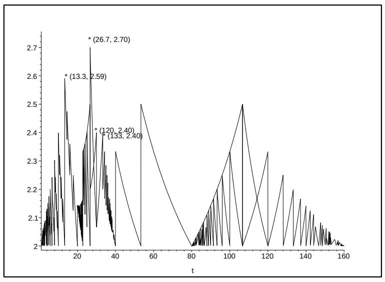

Figure. 4: Strategic point under asynchronous time constraint ($\frac{\tau^1}{\tau^2} = \frac{1}{4}$)

Figure 4 describes that the function of a specific case, here $\frac{\tau^1}{\tau^2} = \frac{1}{4}$, takes its single largest value 2.70 at the one-third shorter remaining duration of decision process. Its peak is a strategic point to minimize cost of time. A strategic point is always located at the one-third shorter remaining duration of decision process in the following range of the ratios of the shorter duration to the longer duration of decision process, $0 < \frac{\tau^1}{\tau^2} < \frac{2}{3}$, but for the two exceptions, as below. First, the function of the specific case, here $\frac{\tau^1}{\tau^2} = \frac{1}{3}$, takes its two largest values 2.63 at the one-third shorter entire duration and the one-third shorter remaining duration of decision process, respectively. Those two peaks are strategic points to minimize cost of time. Those points are always located at the one-third shorter entire duration and the one-third shorter remaining duration of decision process in the following range of the ratios of shorter duration to longer duration of decision process, $0 < \frac{\tau^1}{\tau^2} = \frac{1}{2n+1}, n \in \mathcal{N} \leq \frac{1}{3}$. Second, the function of other specific case, here $\frac{\tau^1}{\tau^2} = \frac{7}{24}$, takes its single largest value 2.60 not at but close to the one-third shorter remaining duration of decision process. Its peak is a strategic point to minimize cost of time. A single strategic point is always located not at but close to the one-third shorter remaining duration of decision process in the following only three cases of the ratios of shorter duration to longer duration of decision process, $\frac{\tau^1}{\tau^2} = \frac{7}{24}, \frac{7}{20}, \frac{7}{19}$. We provide this finding as a theorem as below:

Theorem 3

$$\arg\max_{[0\leq t\leq 1]} \frac{P^a}{C^a_{(t)}} = \arg\max_{[0\leq t\leq 1]} \frac{1}{2} \sum_{\tau\in\tau^1,\tau^2} \frac{P^a_\tau}{C^a_{\tau(t)}}$$

$$= \lim_{t\to 2\frac{\tau^1}{3}\mp 0} \frac{1}{2} \sum_{\tau\in\tau^1,\tau^2} \frac{P^a_\tau}{C^a_{\tau(t)}} = \frac{3}{2} + \frac{3}{2} \cdot \frac{\tau^2}{\tau^1} \frac{1}{\lfloor \frac{3\tau^2}{2\tau^1+0} \rfloor}$$

$$(0 < \frac{\forall\tau^1}{\forall\tau^2} \neq \frac{7}{24}, \frac{7}{20}, \frac{7}{19} < \frac{2}{3}); \tag{10}$$

$$\arg\max_{[0\le t\le 1]} \frac{P^a}{C^a_{(t)}} = \arg\max_{[0\le t\le 1]} \frac{1}{2}\sum_{\tau\in\tau^1,\tau^2} \frac{P^a_\tau}{C^a_{\tau(t)}}$$
$$= \lim_{t\to\frac{\tau^1}{3}+0} \frac{1}{2}\sum_{\tau\in\tau^1,\tau^2} \frac{P^a_\tau}{C^a_{\tau(t)}} = \frac{3}{2} + 3\cdot\frac{\tau^2}{\tau^1}\frac{1}{\lfloor\frac{3\tau^2}{\tau^1+0}\rfloor}$$
$$(0 < \frac{\tau^1}{\tau^2} = \frac{1}{2n+1}, n\in\mathcal{N} < \frac{2}{3}); \quad (11)$$

And,

$$\arg\max_{[0\le t\le 1]} \frac{P^a}{C^a_{(t)}} = \arg\max_{[0\le t\le 1]} \frac{1}{2}\sum_{\tau\in\tau^1,\tau^2} \frac{P^a_\tau}{C^a_{\tau(t)}}$$
$$= \lim_{t\to 2\frac{\tau^1}{3}+\alpha\mp 0} \frac{1}{2}\sum_{\tau\in\tau^1,\tau^2} \frac{P^a_\tau}{C^a_{\tau(t)}}$$
$$= \frac{\tau^1}{\lfloor\frac{\tau^1}{\tau^1-\{(2\frac{\tau^1}{3}+\alpha)-0\}}\rfloor\cdot\{\tau^1-(2\frac{\tau^1}{3}+\alpha-0)\}}$$
$$+\frac{\tau^2}{\lfloor\frac{\tau^2}{\{(2\frac{\tau^1}{3}+\alpha)+0\}}\rfloor\{(2\frac{\tau^1}{3}+\alpha)+0\}} \quad (\frac{\tau^1}{\tau^2} = \frac{7}{24}, \frac{7}{20}, \frac{7}{19}). \quad (12)$$

s.t. $\tau^1 < \tau^2$*;* $0 < \alpha \in \mathcal{R}$ *is given as a certain proper static value.*

Other two cases on the function follow similar theorems. In another range of the ratios of shorter duration to longer duration of decision process, $\frac{2}{3} \le \frac{\tau^1}{\tau^2} \le \frac{\sqrt{6}}{3}$, a single strategic point is always located at the one-third shorter entire duration of decision process but for two exceptions. First, the function at $\frac{\tau^1}{\tau^2} = \frac{\sqrt{6}}{3}$ takes its two largest values 2.72 at the one-third shorter entire duration and the one-third longer entire duration of decision process, respectively. Second, the function at $\frac{\tau^1}{\tau^2} = \frac{2}{3}$ takes its single largest value 2.67 not at but close to the one-third shorter entire duration of decision process. The single strategic point is always located not at but close to the one-third shorter entire duration of decision process in the following range of the ratios of shorter duration to longer duration of decision process, $\frac{2}{3} \le \frac{\tau^1}{\tau^2} < \frac{3}{4}$.

Theorem 4

$$\arg\max_{[0\le t\le 1]} \frac{P^a}{C^a_{(t)}} = \arg\max_{[0\le t\le 1]} \frac{1}{2}\sum_{\tau\in\tau^1,\tau^2} \frac{P^a_\tau}{C^a_{\tau(t)}}$$
$$= \lim_{t\to\frac{\tau^1}{3}+0} \frac{1}{2}\sum_{\tau\in\tau^1,\tau^2} \frac{P^a_\tau}{C^a_{\tau(t)}} = \frac{3}{2} + 3\cdot\frac{\tau^2}{\tau^1}\frac{1}{\lfloor\frac{3\tau^2}{\tau^1+0}\rfloor}$$
$$(\frac{3}{4} \le \frac{\forall\tau^1}{\forall\tau^2} < \frac{\sqrt{6}}{3}); \quad (13)$$

$$\arg\max_{[0\le t\le 1]} \frac{P^a}{C^a_{(t)}} = \arg\max_{[0\le t\le 1]} \frac{1}{2}\sum_{\tau\in\tau^1,\tau^2} \frac{P^a_\tau}{C^a_{\tau(t)}}$$
$$= \lim_{t\to\frac{\tau^2}{3}+0} \frac{1}{2}\sum_{\tau\in\tau^1,\tau^2} \frac{P^a_\tau}{C^a_{\tau(t)}} = 3\cdot\frac{\tau^1}{\tau^2}\frac{1}{\lfloor\frac{3\tau^1}{\tau^2+0}\rfloor} + \frac{3}{2}$$
$$= \frac{3+\sqrt{6}}{2} \quad (\frac{\tau^1}{\tau^2} = \frac{\sqrt{6}}{3}); \quad (14)$$

And,

$$\arg\max_{[0\le t\le 1]} \frac{P^a}{C^a_{(t)}} = \arg\max_{[0\le t\le 1]} \frac{1}{2}\sum_{\tau\in\tau^1,\tau^2} \frac{P^a_\tau}{C^a_{\tau(t)}}$$
$$= \lim_{t\to\frac{\tau^1}{3}+\beta+0} \frac{1}{2}\sum_{\tau\in\tau^1,\tau^2} \frac{P^a_\tau}{C^a_{\tau(t)}}$$
$$= \frac{\tau^1}{\lfloor\frac{\tau^1}{\{(\frac{\tau^1}{3}+\beta)+0\}}\rfloor\{(\frac{\tau^1}{3}+\beta)+0\}}$$
$$+\frac{\tau^2}{\lfloor\frac{\tau^2}{\{(\frac{\tau^1}{3}+\beta)+0\}}\rfloor\{(\frac{\tau^1}{3}+\beta)+0\}} \quad (\frac{2}{3} \le \frac{\forall\tau^1}{\forall\tau^2} < \frac{3}{4}). \quad (15)$$

s.t. $\tau^1 < \tau^2$*;* $0 < \beta \in \mathcal{R}$ *is given as a certain proper static value.*

In the other range of the ratios of shorter duration to longer duration of decision process, $\frac{\sqrt{6}}{3} < \frac{\tau^1}{\tau^2} < 1$, a strategic point of the one-third shorter entire duration of decision process is exceptionally accompanied with another strategic point of the one-third longer entire duration of decision process at $\frac{\tau^1}{\tau^2} = \frac{6}{7}$.

Theorem 5

$$\arg\max_{[0\le t\le 1]} \frac{P^a}{C^a_{(t)}} = \arg\max_{[0\le t\le 1]} \frac{1}{2}\sum_{\tau\in\tau^1,\tau^2} \frac{P^a_\tau}{C^a_{\tau(t)}}$$
$$= \lim_{t\to\frac{\tau^2}{3}+0} \frac{1}{2}\sum_{\tau\in\tau^1,\tau^2} \frac{P^a_\tau}{C^a_{\tau(t)}} = \frac{3}{2}(\frac{\tau^1}{\tau^2}+1) \quad (\frac{\sqrt{6}}{3} < \frac{\tau^1}{\tau^2} < 1).$$

Theorems 3 to 5 describe that strategic points under asynchronous time constraints are located at or close to the one-third shorter entire duration and/or at the one-third shorter remaining duration or at the one-third longer entire but not remaining duration of decision process in the respective ranges of the ratios of shorter duration to longer duration of decision process. In the next section, we apply the above formulas on cost of time and strategic points to a case study.

III. Case Study

In this section, we apply the formulas on the cost of time and the strategic point to the following case study for their feasibility check.

Case: *The Multisearch Software Case* is an introductory practice for decision making in American business schools [10]. That case is a business alliance game on a certain software development between a developer and a company.

Participant record: We have three different groups for their trial sessions. All eight participants of A and B, C and D, E and F, and G and H constitute three separate groups 1 (A, B), 2 (C-D, E-F) and 3 (G, H), respectively:

- Those participants are divided in three groups, two single parties of one-to-one players and a multi-party of two-to-two players; And,
- Respective groups negotiate over a single case once, respectively.

Table 1: The results on the case study.

Company side	A & G	C & D
Developer side	**B**	**E, F & H**
Prices of individual items (*outcomes*)		
1. Royalty to developer	10%, 4 years	8%, 5 years
2. Advance to developer	$250,000-	$150,000-
3. Promotion for sales	$1,000,000-	$1,100,000-
4. The additional to developer	N/A	$1,000,000- (5 years)
5. Commitment by developer to company	$300,000- (2 years)	$750,000- (5 years)
6. Developer's independent gain	$150,000- /year after the 3rd year	N/A
Additive of 1 to 6	$P^{s\,1}$ = $2,400,000-	$P^{s\,2}$ = $3,700,000-
Entire duration (*given*)	τ^1 = 120 minutes	τ^2 = 160 minutes
Selection of strategy	Stay	Stay

Entire duration of decision process: The developer accepts 3 to 6 months and the company does 3 to 4 months for the release of their final product. The individual duration of decision process is scaled down in practice to 120 minutes for A, B and G and 160 minutes for C, D, E, F and H, respectively.
Elapsed time record: The sessions took 50 minutes in Group 1 of A and B, 95 minutes in Group 2 of C-D and E-F and 50 minutes in Group 3 of G and H.
Prices of individual items: Table 1 describes the prices of individual items in detail which are their negotiated outcomes. The estimated annual revenue from the sales of the software product was given as $1,000,000 to those groups in advance. The monetary value of stock option and pension plan was negotiated and agreed as $300,000 in total among those groups in practice.

IV. Discussion

In this section, we discuss the applications of the proposed formulas on cost of time and strategics point with their contributions and limitations. The primary issue is whether the proposed formulas are to suggest any improvement in the decision process of respective groups regarding the results of the case study.
Group 1 passed the first strategic point at 40 minutes and closed its game at the point of 50 minutes, which was even before the heuristic point of half time, 60 minutes, as described in Figure 5. The ratio at 50 minutes was 2.40 (

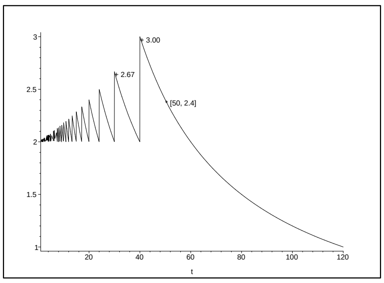

Figure. 5: Group 1's ratio of gain to cost of time (X: t (minutes); Y: $\frac{P^1}{C^1_{(t)}}$): equation (4)

$\because \frac{P^1}{C^1_{(50)}} = \frac{2\cdot 120}{\lfloor \frac{120}{50} \rfloor \cdot 50}$). That ratio of 2.40 was smaller than the ratio of 3.00 at the strategic point and even another ratio of 2.67 at its previous second-best peak point.
Group 2 passed the first strategic point at 53.3 minutes and

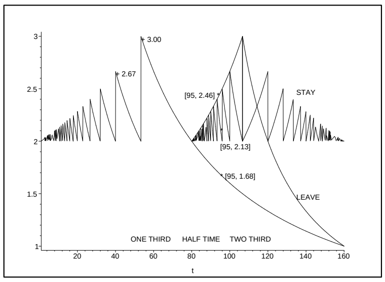

Figure. 6: Group 2's ratio of gain to cost of time (X: t (minutes); Y: $\frac{P^2}{C^2_{(t)}}$): equations (4), (6) and (7)

closed its game at the point of 95 minutes, which was located before the second heuristic point of half time of its remaining duration, *i.e.*, 120 minutes, but quite close to another strategic point of 107 minutes, as described in Figure 6. The ratios at 95 minutes were 1.68 ($\because \frac{160}{95}$) on Equation (4), 2.13 ($\because \frac{160}{\lfloor \frac{160}{2\cdot 95 - 160} \rfloor (95 - \frac{160}{2})}$) on Equation (6) and 2.46 ($\because \frac{2\cdot 160}{\lfloor \frac{160}{160-95} \rfloor (160-95)}$) on Equation (7), respectively. The ratio of 2.46 was smaller than the ratio of 3.00 at the strategic point and even another ratio of 2.67 at its previous second-best peak point.

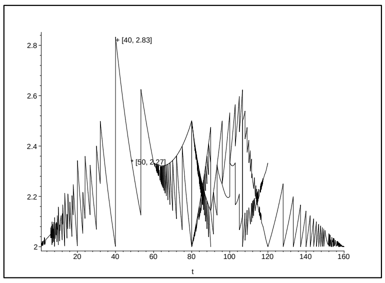

Figure. 7: Group 3's ratio of gain to cost of time (X: t (minutes); Y: $\frac{P^3}{C^3_{(t)}}$): equation (9)

Group 3 passed the first strategic point at 40 minutes and closed its game at the point of 50 minutes, which was before the heuristic point of half time or the half shorter entire duration, 60 minutes, as described in Figure 7. The ratio at 50 minutes was 2.27 ($\because \frac{P^3}{C^3_{(50)}} = \frac{120}{\lfloor \frac{120}{50} \rfloor \cdot 50} + \frac{160}{\lfloor \frac{160}{50} \rfloor \cdot 50}$). That ratio of 2.27 was the smaller than the ratio of 2.83 at the strategic point.
The participants faced Equations (1) and (3) as their respec-

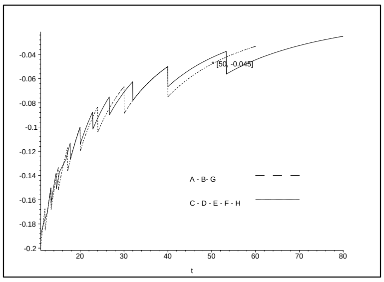

Figure. 8: The differentiated ratio of gain to cost of time before the half time (X: t (minutes); Y: $\frac{d\frac{P}{C_{(t)}}}{dt}$): the differentiation of Equation (4)

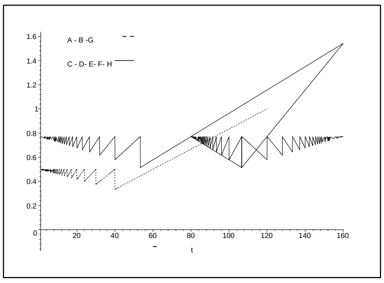

Figure. 9: Cost of time (X: t (minutes); Y: $C_{(t)}$): Equations (1) and (3); P^{1s} is given as 1

tive functions on cost of time, as described in Figure 9. Here, A, B and G's gain as their negotiated outcomes or the additives of prices of individual items in their single stage games, P^{1s}, was given as 1 so that C, D, E, F and H's gain in their single stage game was to be 1.54 ($\because \frac{3,700,000}{2,400,000}$). The initial value on C, D, E, F and H's cost of time was evaluated as larger than the initial value on A, B and G's cost of time; C, D, E, F and H's realized gain in their single stage games was larger so that their initial object for gain was evaluated as the larger than the initial object for A, B and G's gain.

We discuss the elements on the selection of the strategic points under synchronous and asynchronous time constraint. First, the entire duration of decision process is to be long enough. Second, larger gain to longer duration allows decision makers to face the more appropriate initial value in the function on cost of time. C, D, E and F realized the larger amount of gain in proportion to the longer duration (2.31 $\times\ 10^4 = \frac{\$3,700,000}{160 \text{ minutes}}$) rather than A and B did (2.00 $\times 10^4 = \frac{\$2,400,000}{120 \text{ minutes}}$) so that C, D, E and F enjoyed the appropriate proportion of the realized gain to the cost of time. Third, better timing allows decision makers to face a variety of options from the first strategic point to the second strategic point, if possible, even after the first heuristic point of the half time. Fourth, decision makers are to improve their decision making with their preliminary communication on their uncommon time constraint. G and H were quite close to their single strategic point. Finally, cost of time increases under asynchronous time constraint rather than under synchronous time constraint, ($\frac{P^a}{C^a_{(\frac{\tau^1}{3})}} = 2.83 < \frac{P^1}{C^1_{(\frac{\tau^1}{3})}} = 3.00$), as described in Figure 7. The synchronization of time constraint allows decision makers to always enjoy two strategic points and to decrease the cost of time.

The contributions of the proposed formulas are found, as below. First, the monetary evaluation of cost of time allows transparent decision making under time constraint. Second, strategic points allow decision makers to take advantages of time resources more effectively, instead of the heuristic point of half time. Third, strategic points allow decision makers to mutually communicate on and to amicably synchronize their duration of decision process for accelerating time-sensitive decision making. The remaining limitations to this study are found, as below: First, the proposed formulas still accept given initial values on prices of individual items; And, Second, strategic point for multilateral decision making under asynchronous time constraint is the next problem for our future work.

V. Conclusion

In this study, we have proposed a mathematical foundation for bilateral decision making under synchronous and asynchronous time constraint. We discussed the problem on time constraint and proposed formulas to compute cost of time by introduction of opportunity cost to its evaluation under time constraint. We found strategic points which always exist at or close to the one-third shorter entire and/or at the one-third shorter remaining duration or at the one-third longer entire but not remaining duration of decision process in respective ranges of the ratios of shorter duration to longer duration of decision process. Strategic points accelerate time-sensitive decision making which is indispensable to any solutions for optimization in intelligent computing, instead of the heuristic point of half time. We have conducted the feasibility check on the proposed formulas in their applications to a case study. In our future work, we would extend our proposed formulas to the multilateral decision making under asynchronous time constraint and implement it in agent-based intelligent systems.

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References

[1] O. Ashenfelter, M. Greenstone. Using Mandated Speed Limits to Measure The Value of A Statistical Life, *J. Pol. Econ.* CXII (2), part 2, pp. 226-267, 2004.

[2] S. Ba, A. B. Whinston, H. Zhang. The Dynamics of The Electronic Market: An Evolutionary Game Approach, *Inf. Syst. Front.* II (1), pp. 31-40, 2000.

[3] D. G. Baird, R. H. Gertner, R. C. Picker, *Game Theory and the Law*, 3rd Ed. Harvard University Press, Cambridge, 1998.

[4] E. Berne. *Games People Play: The Psychology of Human Relationships*. Grove Press, New York, 1964.

[5] M. Bichler. Trading Financial Derivatives on The Web: An Approach towards Automating Negotiations on OTC Markets, *Inf. Syst. Front.* I (4), pp. 401-414, 2000.

[6] J. H. Chen, K. M. Chao, N. Godwin, V. W. Soo. Combining Cooperative and Non-Cooperative Automated Negotiations, *Inf. Syst., Front.* VII (4/5), pp. 391-404, 2005.

[7] Z. Cheng, M. A. M. Capretz, M. Osano. A Model for Negotiation among Agents Based on The Transaction Analysis Theory. In *Proceedings of the Symp. Autonomous Decentralized Syst.*, pp. 427-433, 1995.

[8] I. E. Dror, J. R. Busemeyer, B. Basola. Decision Making under Time Pressure: An Independent Test of Sequential Sampling Models, *Memory and Cognition* XXVII (4), pp. 713-725, 1999.

[9] J. K. Goeree, C. A. Holt, T. R. Palfrey. Risk Averse Behavior in Asymmetric Matching Pennies Games, *Games and Econ. Behavior* XLV (1), pp. 97-113, 2003.

[10] E. C. Gould, M. Easter. *The Multisearch Software Case*. Program on Negotiation at Harvard Law School, Cambridge, 1998.

[11] S. Greenstein. The Commercialization of Information Infrastructure as Technological Mediation: The Internet Access Market, *Inf. Syst. Front.* I (4), pp. 329-348, 2000.

[12] L. Kriesberg. Timing and The Initiation of De-Escalation Moves, *Nego. J.* III (4), pp. 375-384, 2007.

[13] M. S. Lee, B. T. Ratchford, D. Talukdar. The Impact of The Internet on Information Search for Automobiles, *Rev. Marketing Sci.* I (2), Working Paper 1, pp. 1-47, 2002.

[14] P. Lehner, M.-M. Seyed-Solorforough, M. F. O'Connor, S. Sak, T. Mullin. Cognitive Biases and Time Stress in Team Decision Making, *IEEE Trans. Syst., Man and Cybernetics, Part A: Syst. and Humans* XXVII (5), pp. 698-703, 1997.

[15] S. Noh, P. J. Gmytrasiewicz. Flexible Multi-Agent Decision Making under Time Pressure, *IEEE Trans. Syst., Man and Cybernetics, Part A: Syst. and Humans* XXXV (5), pp. 697-707, 2005.

[16] L. A. O'Grady, H. Witteman, C. N. Wathen. The Experiential Health Information Processing Model: Supporting Collaborative Web-Based Patient Education, *BMC Med. Informatics and Dec. Making* VIII (58), pp. 1-22, 2008.

[17] D. L. Olson. Rationality in Information Systems Support to Decision Making, *Inf. Syst. Front.* III (2), pp. 239-248, 2001.

[18] K. M. Osei-Bryson, O. Ngwenyama. Decision Models for Information Systems Management, *Inf. Syst. Front.* X (3), pp. 277-279, 2008.

[19] J. W. Payne, J. R. Bettman, M. F. Luce. When Time Is Money: Decision Behavior under Opportunity Cost Time Pressure, *Organizational, Behavior and Human Dec. Proc.* LXVI, pp. 131-152, 1996.

[20] S. Russell, E. Wefald. On Optimal Game-Tree Search Using Rational Meta-Reasoning. In *Proceedings of the Joint Conf. on A.I.*, pp. 334-340, 1989.

[21] H. Sasaki. An Evaluation Method for Strategic Decision Making on Group Collaboration under Temporary Constraints. In *Proceedings of the IEEE Symp. Adv. Magn. of Info. for Globalized Enterprises (AMIGE)*, no. 10, pp. 1-5, 2008.

[22] H. Sasaki. Strategic Decision Making on Group Collaboration under Temporary Constraints. In *Proceedings of the IEEE/ACM Conf. Soft Comp. as Transdisciplinary Sci. and Tec. (CSTST)*, pp. 343-349, 2008.

[23] H. Sasaki. A Computing Theory for Collaborative and Transparent Decision Making under Time Constraint, *Inf. Syst. Front.* XIII (2), pp. 207-220, 2011.

[24] H. Sasaki. Decision Making under Time Constraint: From Heuristics to Strategy. In *Proceedings of the IEEE Int'l. Conf. Syst., Man, and Cybernetics (SMC)*, 2010, pp. 2935-2940.

[25] H. Sasaki. Strategic Points for Decision Making under Time Constraint. In *Proceedings of the Int'l Conf. P2P, Parallel, Grid, Cloud and Internet Computing (3PGCIC)*, Silver Spring: IEEE Computer Society Press, pp. 308-313, 2010.

[26] H. Sasaki. A Non-Heuristic Model of Decision-Making Based on Human Perception under Time Constraints. In *Proceedings of the IEEE Int'l. Conf. Syst., Man, and Cybernetics (SMC)*, pp. 621-626, 2011.

[27] E. Schragenheim. A Systematic Approach to Common and Expected Uncertainty, *J. Syst. Improvement* I (2), pp. 1-8, 1997.

[28] R. S. Sutton, A. G. Barto. *Reinforcement Learning: An Introduction*. MIT Press, Cambridge, 1998.

[29] N. Szirbik. A Negotiation Enabling Agent Based Infrastructure: Composition and Behavior, *Inf. Syst. Front.*, IV (1), pp. 85-99, 2002.

[30] M. W. H. Weenig, M. Maarleveld. The Impact of Time Constraint on Information Search Strategies in Complex Choice Tasks, *J. Econ. Psy.*, XXIII (6), pp. 689-702, 2002.

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